Abstract—This paper presents a new method for automatic segmentation of thermographic frontal breast images. First, the method uses a low-pass filter and the topological derivative to get a rough definition of the region of interest (ROI). Then, the previous result is used to initialize the method proposed by Marques. This method uses the adjustment of curves by Quadratic Uniform B-Splines. The efficiency of the proposed method is quantitatively verified using a ground truth developed by a group of experts. Besides, we provide comparisons with a related technique.

I. INTRODUCTION

Breast cancer is the second most frequent type of cancer in the world and the most common among women. Its rate is of 22% of new cases each year [1]. However, in case of early detection and appropriately treatment, the prognosis is good. Since breast cancer is the most common kind of cancer among Brazilian women and women of the world new researches, technologies focused on solving or even minimizing these facts are of great importance for medicine and society. Moreover, the use of thermal imaging could improve this early diagnosis in four years [2].

For computer-aided diagnostic systems, the first step after acquiring the thermal image is to segment it to obtain a region of interest (ROI). The ROI segmentation intends to separate the regions of the breast from the other parts of the image. It can range from a completely manual to a fully automatic process. The breast, also called the mamma, is made up of connective tissue, fat, and breast tissue that contain the glands that can make milk. The mammary gland is made up of lobes (or portion of breast) and ducts (a tube through which the milk passes). Each breast has 15 to 20 sections called lobes, which have many smaller sections called lobules (subdivision of a lobe). Lobules end in dozens of tiny bulbs that can produce milk. The lobes, lobules, and bulbs are linked by thin tubes called ducts. Each breast also has blood and lymph vessels. The lymph vessels carry almost colorless fluid that carries cells that help fight infections and other diseases: the lymph. Lymph vessels lead to a rounded mass of lymphatic tissue that is surrounded by a capsule of connective tissue: the lymph nodes, that filter lymph (lymphatic fluid), and they store lymphocytes (white blood cells). They are located along lymphatic vessels and called lymph gland, as well. Lymph nodes are small bean-shaped structures that are found throughout the body. They filter substances in lymph and help fight infections and diseases. Clusters of lymph nodes are found near the breast in axillary region (that is under the arm), above the collarbone, and in the chest. For this the ROI must include these regions. That is the extraction from the acquired images of the regions of interest (ROI) must include all breast tissues and the near ganglion groups related as much as possible. Some medical doctors most concerned with thermal analysis also suggest the inclusion of half of the arm in it. About 75% of lymph from the breasts drains into the axillary lymph nodes, making them important in the diagnosis of breast cancer. The extraction of the ROI limits on the infrared images is a very hard task due to their amorphous nature and lack of clear limits. Some authors prefer manual segmentation but for Computer-Aided Diagnosis(CADx) and Computer-Aided Detection(CADe) systems it must be done automatically.

There are several papers in the area of thermographic image segmentation. Lipari and Head [3] propose a protocol of segmentation where each breast is divided into four quadrants. The breasts are divided by four reference points: the chin, the left side, the right side and the bottom edge of the breast. Each of these points connected to the nipple separates the image into four distinct quadrants. Herry and Frize [4] have done statistical analysis and a comparison of the intensity distribution on both mammas. Then their classification was done by extracting the contours using a simple contour detector and morphological operations. However, the authors state that the segmentation must be done manually. Zhou et al. [5] propose the use of Level Set Method (LSM) to extract the edges of an object in a thermal image. The approach used is based on the direction and magnitude of the edge. Edge Map represents the gradient magnitude and direction. To obtain the Edge Map linear and isotropic features based on Gaussian filter are used. Three or four randomly selected points within the region of interest (or on its edge) are used as initial points. The initial pixels locate the place where the evolution of the method begins and provides information of the gradient.

Recently, the method that uses BSP, proposed by Marques [6], was presented which includes the following steps: lateral segmentation, upper bounds segmentation and lower bounds segmentation. The lateral segmentation step consists on the detection of the patient body borders. The body borders are obtained using Otsu’s Method and Laplacian filter. After that, the upper bounds are found through the detection of shoulders or raised arms. The lower bounds detection is the most important step in Marques’ methodology. This final step is focused on the detection of the inframammary folds. To accomplish...
that, the author makes use of thresholding techniques, region growing, clusterization based on pixels neighborhood and a series of heuristic refinements to find a set of initial points corresponding to the inframammary fold. After that, those initial set of points are smoothed by a Uniform Quadratic Basis Spline, finalizing the method.

Despite of the advantages of BSP we observe that pre-segmentation remains a problem due to the complex intensity field of thermographic images. On the other hand, topological derivative has been successfully used for pre-processing medical images [7]. That is our motivation for applying topological derivative for thermographic images. The concept of topological derivative was initially conceived to deal with topology optimization problems [8]. In the last decade, it was applied for image processing [9].

Until now, the few studies ([3], [4], [5]) that have automatically detected the ROI in breast thermograms did not present the validation of their results. The correct extraction of the region of interest is essential for further steps in image analysis process. Therefore, the validation of our proposal will be quantitatively checked by using Hausdorff distance and the computation of accuracy, sensitivity, specificity, positive predictive and negative predictive [10].

This paper develops a new methodology based on topological derivative for the extraction of the region of interest (ROI) of the breast in thermographic images. The proposed methodology is composed by the following steps: (a) Average Filter; (b) Topological Derivative; (c) Quadratic Uniform B-Splines (BSP)[6].

II. TOPOLOGICAL DERIVATIVE IN IMAGE SEGMENTATION

Let us consider an open bounded domain \( \Omega \subset \mathbb{R}^d \), with \( d \geq 2 \), subject to a non-smooth perturbation confined in a small region \( \omega_\varepsilon(\hat{x}) = \hat{x} + \varepsilon \omega \) of size \( \varepsilon \), where \( \hat{x} \) is an arbitrary point of \( \Omega \) and \( \omega \) is a set of points in \( \mathbb{R}^d \), as shown in Fig. 1.

![Fig. 1. The topological derivative concept.](image)

Associated to the original (unperturbed) domain, \( x \rightarrow \chi(x) \), \( x \in \mathbb{R}^d \), we have \( \chi = 1_\Omega \) (notation characteristic function), such that

\[
|\Omega| = \int_{\mathbb{R}^d} \chi,
\]

where \( |\Omega| \) is the Lebesgue measure of \( \Omega \). Then,

\[
\chi = 1_\Omega = \begin{cases} 1 & \text{se} \ x \in \Omega, \\ 0 & \text{se} \ x \in \mathbb{R}^d \setminus \Omega, \end{cases}
\]

Moreover, associated to the topologically perturbed domain, we defined the characteristic function of the shape \( x \rightarrow \chi_\varepsilon(\hat{x}; x), \hat{x} \in \mathbb{R}^d \). In the case a perturbed non smooth, for example, this function is defined as, \( \chi_\varepsilon(\hat{x}) = 1_{\Omega_\varepsilon} - (1 - \gamma)1_{\omega_\varepsilon(\hat{x})} \), where \( \gamma \) is the contrast in the intensity of the source term.

\[
\gamma_\varepsilon(x) = \begin{cases} 1 & \text{se} \ x \in \Omega_\varepsilon \setminus \omega_\varepsilon(\hat{x}), \\ \gamma & \text{se} \ x \in \omega_\varepsilon(\hat{x}), \end{cases}
\]

with \( \gamma \in \mathbb{R} \) and \( \chi_\varepsilon(\hat{x}) = \gamma_\varepsilon \).

We assume that a given shape functional \( \psi(\chi_\varepsilon(\hat{x})) \), associated to the topologically perturbed domain, admits the following topological asymptotic expansion

\[
\psi(\chi_\varepsilon(\hat{x})) = \psi(\hat{x}) + f(\varepsilon)D_\varepsilon(\hat{x}) + o(f(\varepsilon)),
\]

where \( \psi(\hat{x}) \) is the shape functional associated to the original (unperturbed) domain, \( f(\varepsilon) \) is a positive function such that \( f(\varepsilon) \to 0 \), when \( \varepsilon \to 0 \). The function \( \hat{x} \mapsto D_\varepsilon(\hat{x}) \) is called the topological derivative of \( \psi \) at \( \hat{x} \). Therefore, this derivative can be seen as a first order correction of \( \psi(\hat{x}) \) to approximate \( \psi(\chi_\varepsilon(\hat{x})) \). More precisely, the topological derivative \( D_\varepsilon(\hat{x}) \) is a scalar function defined over the original domain that indicates, in each point, the sensitivity of the shape function when a singular perturbation of size \( \varepsilon \) is introduced at that point. In general, the domain singular perturbation can be, for instance: the introduction of holes, cracks or non smooth changes in the parameters of the problem (e.g., material properties, sources acting over the domain, boundary conditions, etc.).

Among the methods for calculation of the topological derivative currently available in literature, here we shall adopt the methodology developed in [11], which is based on the following result

\[
D_\varepsilon(\hat{x}) = \lim_{\varepsilon \to 0} \frac{1}{f(\varepsilon)} \frac{d}{d\varepsilon} \psi(\chi_\varepsilon(\hat{x})),
\]

where \( \frac{d}{d\varepsilon} \psi(\chi_\varepsilon(\hat{x})) \) is the derivative of \( \psi(\chi_\varepsilon(\hat{x})) \) with respect to the small parameter \( \varepsilon \), which can be seen as the sensitivity of \( \psi(\chi_\varepsilon(\hat{x})) \), in the classical sense [12], [13], to the domain perturbation produced by an uniform expansion of the perturbation \( \omega_\varepsilon \), as shown in Fig. 2, in other words, \( \omega_\varepsilon + \varepsilon \chi \omega_\varepsilon = \omega_\varepsilon(\hat{x}) + \varepsilon \omega_\varepsilon \).

![Fig. 2. Uniform expansion of the perturbation \( \omega_\varepsilon \).](image)

Therefore, we can use the concept of shape sensitivity analysis as an intermediate step in the topological derivative calculation. We will see later that this procedure enormously simplifies the analysis.

A. Problem Formulation

In [9], a shape function that quantifies the misfit between the input image \( v \) being segmented and a possible segmentation \( u \) has been proposed. Let us first define the input image \( v \) as

\[
v \in V = \{ v \in L^2(\Omega) : v \text{ constant by parts} \},
\]
and the segmented image \( u \) as
\[
u \in U = \{ u \in V : u(\mathbf{x}) \in C, \forall \mathbf{x} \in \Omega \}, \tag{7}\]
where \( \Omega \) the set of classes \( C \) is given by
\[
C = \{ c_i \in \mathbb{R} : i = 1, \ldots, N_c \}, \tag{8}\]
with \( N_c \) used to denote the number of classes in which the original image \( v \) will be segmented and \( c_i \) represents the intensity that characterizes the \( i^{th} \)-class. Therefore, let us introduce the following shape function, which is inspired on the Mumford-Shah functional [14]
\[
\psi(\chi) := J(\varphi) = \frac{1}{2} \int K \nabla \varphi \cdot \nabla \varphi + \frac{1}{2} \int (\varphi - (v - u))^2, \tag{9}\]
where the field \( \varphi \) accounts for the misfit between \( v \) and \( u \), and is solution of the following variational problem: find \( \varphi \in H^1(\Omega) \), such that:
\[
\int \nabla \varphi \cdot \nabla \eta + \int \varphi \eta = \beta \int (v - u) \eta \quad \forall \eta \in H^1(\Omega). \tag{10}\]
The diffusive second order tensor field \( K \) is constant at image element level and \( 0 < \beta \leq 1 \) is used to adjust the numerical algorithm. Note that the segmented image \( u \) and function \( \varphi \) can be seen as the control and the state, respectively.

Therefore, the image segmentation problem can be stated as following: given the image data \( v \in V \), find the segmented image \( u^* \in U \) such that minimizes the functional (9) when choosing successively the class that produces the most negative value of the topological derivative.

### B. Topological Derivative Calculation

Associated to \( \varphi \) we define the function \( \varphi_\varepsilon \) solution to the perturbed variational problem. In this context, the perturbation is characterized by changing the segmented image \( u \) for a new one \( u_T \) that is identical to \( u \) everywhere in \( \Omega \) except in a small region \( \omega_\varepsilon \) centered in \( \mathbf{x} \in \hat{\Omega} \). In \( \omega_\varepsilon \), \( u_T \) assumes one of the values \( c_i \in C \). We work with inclusion, in which a change in the forces/sources that act in \( \Omega \), the characteristic function \( \chi_\varepsilon \) is defined as above, where \( c_i \) is the contrast in the intensity of the source term.
\[
\chi_\varepsilon(\mathbf{x}) = 1_{\Omega} - (1 - c_i)1_{\omega_\varepsilon}(\mathbf{x}), \tag{11}\]
More precisely,
\[
u_T = \chi_\varepsilon u = \begin{cases}
u & \text{se } \mathbf{x} \in \Omega \setminus \omega_\varepsilon, \\ c_i & \text{se } \mathbf{x} \in \omega_\varepsilon,\end{cases} \tag{12}\]
In this way, the perturbed shape function becomes
\[
\psi(\chi_\varepsilon) := J(\varphi_\varepsilon) = \frac{1}{2} \int K \nabla \varphi_\varepsilon \cdot \nabla \varphi_\varepsilon + \frac{1}{2} \int (\varphi_\varepsilon - (v - u_T))^2, \tag{13}\]
where the field \( \varphi_\varepsilon \) is the solution to the perturbed variational problem: find \( \varphi_\varepsilon \in H^1(\Omega) \) such that
\[
\int \nabla \varphi_\varepsilon \cdot \nabla \eta + \int \varphi_\varepsilon \eta = \beta \int (v - u_T) \eta \quad \forall \eta \in H^1(\Omega). \tag{14}\]
The associated topological derivative can be easily calculated, namely (see [7] for details),
\[
D_{\Omega}(\varepsilon) = -\frac{1}{2}[\omega(\varepsilon - c_i)][(\varphi(\mathbf{x}) - (v - u)) + (\varphi(\mathbf{x}) - (v - c_i)) + 2(1 - \beta)\varphi(\mathbf{x})], \tag{15}\]
where function \( f(\varepsilon) = \varepsilon^2 \) and \( |\omega| \) is the Lebesgue measure (area) of the set \( \omega \).

### III. PROPOSED APPROACH

The proposed segmentation methodology is composed by the following steps: (a) Average Filter; (b) Topological Derivative; (c) Quadratic Uniform B-Splines (BSP)[6].

Medical images have a complex intensity field and texture patterns. Thus, the low-pass filtering (a) is applied to smooth the original image before using the topological derivative.

As mentioned before, for the image \( v \in V \) we need to find the segmented image \( u^* \in U \) that minimizes the shape function \( J(\varphi) \) by successively selecting the class that produces a negative value of the topological derivative. In this way, the following image segmentation algorithm was proposed in [9] based on the topological derivative.

**Algorithm 1** Image segmentation based on the topological derivative

**Require:** An input image \( v \in V \), the set \( C \), an initial guess \( u \in U \), the diffusivity tensor field \( K \) and the parameters \( \beta \) and the step size \( \alpha \in (0, 1) \).

**Ensure:** The segmented image \( u^* \in U \).

while \( D_{\Omega}(\chi) < 0 \)

\[
\begin{align*}
\text{find the solution } \varphi \text{ to the variational problem (10)} & \quad \text{evaluate } D_{\Omega}(\chi) = D_{\Omega}(\chi, c_i) \text{ according to (15)} \\
\text{compute } c^*(\chi) = \arg \min_{c_i \in C} \{ D_{\Omega}(\chi, c_i) \} & \quad \text{compute } d_T = \min_{\chi \in \Omega} \{ D_{\Omega}(\chi, c^*(\chi)) \} \\
\text{if } (D_{\Omega}(\chi, c^*(\chi)) \leq (\alpha)d_T, \text{ set } u(\chi) = c^*(\chi) & \quad \text{update the class } C \text{ according to Algorithm 2} \\
\end{align*}
\]
end while

\[
u = u^*
\]

The solution \( \varphi \) in Algorithm 1 is obtained by the standard finite element method, where the bilinear elements coincide with the image pixels.

Obviously, a fundamental question is how to define the set of classes \( C \). This is performed by calling the Algorithm 2, after each interaction of the main loop [9]. Basically, the Algorithm 2 takes an initial guess \( C \) (minimum and maximum intensity values for two-class case) and computes \( C^* \). Then, the set \( C \) that minimizes the functional \( J(\varphi) \) in expression (9) is denoted by \( C^* \).

**Algorithm 2** Adjust the values of the classes

**Require:** An input image \( v \in V \), the set \( C \) and the segmented image \( u \in U \) obtained at each iteration of Algorithm 1.

**Ensure:** The new set of classes \( C^* \).

\[
C^* = \begin{cases}
\text{for } c_i \in C \text{ do} & C^* = \begin{cases}
C & \text{for } c_i \in C \\
\text{set } C^* = C^* \cup \{ c_i \} & C^* = C^* \cup \{ c_i \}
\end{cases}
\end{cases}
\]
end for
\[
u = u^*
\]
The obtained result is used to initialize the method BSP and can be seen as an approximation of the boundaries of the targets.

IV. COMPARING ROI AUTOMATIC DETECTION APPROACHES

The images used for comparison are acquired by a Flir Thermacam S45 camera. These images are public available on [15]. This image database has been taken from volunteers that have signed a consent form allowing the use of their images [16]. This acquisition were approved by the Ethical Committee of the Brazilian Ministry of Health. The Fig. 3 is an example of the images stored in this database.

![Original Image](image)

**Fig. 3.** Original image.

For quantitative comparison between each approach two methods are used: the Hausdorff distance and statistics based on region analysis.

The evaluation based in Hausdorff distance (see [6], for details) consists of calculating the distance between the extracted ROI and it’s ground truth. Small distances indicate that the automatic segmentation is close to the ground truth and so the method performs well. The Fig.4.(a) shows the overlapping of the automatic segmentation and the corresponding ground truth.

![Overlapping Segmentation](image)

**Fig. 4.** (a) Overlapping automatic segmentation (dark gray) and ground truth (light gray). (b) Definition of the TP, TN, FP and FN areas used for numerical evaluation of the ROI segmentation, by the green, blue, red and orange, respectively.

The statistical evaluators of accuracy, sensitivity, specificity, positive predictive and negative predictive were estimated using these FP, FN, TP and TN values. The accuracy (ACC) is the proportion of true results (both true positives and true negatives) in the pixels ROI. It is measured by (16):

\[
ACC = \frac{TP + TN}{FN + FP + TN + TP}
\]  

Sensitivity (SEN) or recall measures the proportion of positive outcomes that are correctly identified as such and it is measured by (17):

\[
SEN = \frac{TP}{TP + FN}
\]  

Specificity (ESP) measures the proportion of negative results that are correctly classified as such, i.e., that is do not part of ROI. The referred evaluation were calculated by the equation (18):

\[
ESP = \frac{TN}{TN + FP}
\]  

The calculation of predictive positive (or negative) aims to show the hit ratio positive (or negative) relative to the total ratings positive (or negative) provided by the algorithm. The referred evaluations were calculated by the equations (19,20):

\[
PDP = \frac{TP}{TP + FP}
\]

\[
PDN = \frac{TN}{TN + FN}
\]

Now, comparison of results achieved by approaches can be grade with numerical evaluators.

V. RESULTS

In this section present the numerical results on twenty real breast segmentation using two different approaches. We start by considering gray images of size 320 × 240 pixels as shown in Fig. 3 (section IV). For the proposed approach (section III) filter the original image of Fig.3 to obtain the result shown in Fig.5.(a). Then, this image is pre-segmented by the topological derivative and the result pictured in Fig. 5.(b). This image is input for Quadratic Uniform B-Splines(BSP) method presented in [6].

![BSP Result](image)

**Fig. 6.** (a) BSP results and Fig.6.(b) the result of the proposed approach. There is not a visual significant difference between every segmentations, this proves the need of numerical comparison. For the ground truth generation three manual segmented images for each patient were used. They were manually defined using a Samsung Galaxy P7510 tablet.
were shown in Table I.

Fig. 6. (a) Result of BSP method. (b) Result of proposed approach.

with stylus pen by a specialist in breast radiology and two trained users. A specialized software was developed for it.

To facilitate the comparisons, all three manual segmented results were combining to form a unique ground truth using the voting policy proposed by Li et al. [18].

First, both approaches are compared with the ground truth using the Hausdorff distance. Table I presents results of the proposed approach and BSP method for each image. The proposed approach presents better results for 15 images.

Table II below shows the statistical measures of all results were shown in Table I.

Although this is a good indication that the proposed approach is better than the BSP method we also compare both approaches with the ground truth according to the criteria of accuracy, sensitivity, specificity, positive and negative predictive, as shown in Table III. The Table IV shows some statistics about these results.

**TABLE III. EVALUATION BASED IN AREA BETWEEN THE GROUND-TRUTH AND TWENTY SEGMENTED IMAGES (VALUES OBTAINED IN COLUMNS ARE MULTIPLIED BY 10^{-4})**

**TABLE IV. STATISTICAL MEASURES BETWEEN THE GROUND-TRUTH AND TWENTY SEGMENTED IMAGES.(VALUES OBTAINED IN COLUMNS ARE MULTIPLIED BY 10^{-4})**

It is important to note that both approaches prioritize the segmentation that respects the lower limits of the curvature of the breasts of patients where there is a greater complexity.

So this results were similar being that those from in proposed approach showed a significantly lower standard deviation in terms of specificity. Therefore, in general, proposed approach outperforms the BSP method.

**VI. CONCLUSION**

This work presents a new approach for ROI extraction based on topological derivative and compare this with other approach to image segmentation to support the early diagnosis of breast disease. The extraction of the region of interest in thermal images is a very important first step for the development of CADe and CAxD system.

The quality of segmentation was evaluated qualitatively and quantitatively. The performance of all approaches was evaluated considering their accuracy, sensitivity, specificity,
positive predictive and negative predictive. The Hausdorff distance is used for a numerical evaluation, as well. According to the evaluations statistical measures the proposed approach yielded excellent results, proving to be an effective technique.

It is possible to conclude that the key step for the success of automatic segmentation is to identify the correct set of points related to inframammary folds [6].

With the proposed automatic segmentation it is possible to develop a preliminary CAD system, in order to provide better visualization of thermal imaging assisting in the interpretation of these images and editing reports from patients. Moreover, in this system, it is evident the importance and practicality provided by automatic segmentation of breast region.

ACKNOWLEDGMENT

The authors thank to CAPES for financial support (projects PROENG PE021/2008 and ProCad no. 540/2009) and to UFF-Telemedicine Group an Associated Laboratory of INCT-MACC.

REFERENCES